Masterclass: Cyclic Homology

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Syllabus

Cyclic cohomology theory is a very important part of noncommutative geometry. It can be regarded as the noncommutative analogue of the de Rham homology of currents on a smooth manifold. Connes' Chern character relates cyclic cohomology (resp. homology) to K-homology (resp. K-theory).

This lecture series will include basic theory of cyclic homology, including its applications to topological and algebraic K-theory, and Hopf-cyclic theory, with examples. One goal of these lectures is to prove an index theorem of Connes which is the prototype of index theorems formulated in the context of noncommutative geometry. Time permitting, more recent applications of cyclic theory to issues related to topological insulators and the bulk-boundary correspondence in condensed matter physics will be covered.

- Simplicial and cyclic modules, Hochschild and cyclic complexes and their (co)homology, cyclic cocycles around us, geometric examples.
- Connes' long exact sequence, Connes' spectral sequence, sample computations.
- Stability, homotopy invariance, excision in H-unital case, products.
- Chern character in algebraic K-theory, topological K-theory and K-homology, Connes' index theorem for Fredholm modules, how to prove a number is an integer, examples of index theorems.
- Hopf-cyclic cohomology, relation to Lie algebra and group cohomology.
- Bellisards's work on quantum Hall effect, what is a topological insulator, bulk-boundary correspondence, what cyclic cohomology can teach us about insulators.

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